1. (24 %) $A=\begin{bmatrix}5 & 2\\ 2 & 2\end{bmatrix}$,

(a) (6%) find the eigenvalues and eigenvectors of A and A^{-1}, respectively.

(b) (6 %) calculate A^7 and A^{-7}.

(c) (6%) find the eigenvalues and eigenvectors of A^7 and A^{-7}, respectively.

(d) (6 %) for any vector $x=\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, show that $x^T A^7 x$ and $x^T A^{-7} x$ are always positive unless x is a zero vector.

2. (6%) Evaluate the integral $\int_{(0,1)}^{(1,0)}(4x^3-3x^2y^2)dx-2x^3ydy$ along the path $x^2+y^2=1$.

3. (40%). Let $\tilde{f}(s)=\mathcal{L}[f(t)]=\int_0^\infty f(t)e^{-st}dt$ be the Laplace transform of $f(t)$.

(a). (5%). Find out the Laplace transform of the function $\frac{1}{2}t^2$.

(b). (10%). Show that

$$\mathcal{L}[f(t-a)H(t-a)]=e^{-as}\tilde{f}(s),$$

where $a>0$ is a constant and $H(t)$ is the Heaviside step function.

(c). (10%). Consider the following initial boundary value problem

$$\begin{aligned}
&\tfrac{\partial^2 u}{\partial t^2}(x,t)=\tfrac{\partial^2 u}{\partial x^2}(x,t)+1, \quad 0<x<\infty, \quad t>0,\\
&u(0,t)=0, \quad t>0,\\
&|u(x,t)| \text{ is bounded as } x\to\infty, \quad t>0,\\
&u(x,0)=0, \quad x>0,\\
&\tfrac{\partial u}{\partial t}(x,0)=0, \quad x>0.
\end{aligned}$$

Define

$$\tilde{u}(x;s)=\int_0^\infty u(x,t)\,e^{-st}dt.$$

Using the technique of Laplace transform to show that

$$\begin{aligned}
&\frac{d^2\tilde{u}}{dx^2}(x;s)-s^2\tilde{u}(x;s)+\frac{1}{s}=0,\\
&\tilde{u}(0;s)=0, \quad |\tilde{u}(x;s)| \text{ is bounded as } x\to\infty.
\end{aligned}$$

Note that here the variable s is treated as a constant.

(d). (10%). Find the solution $\tilde{u}(x;s)$ in (c).

(e). (5%). Using (a) and (b) to find the solution $u(x,t)$ of the initial boundary value problem (c).

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4. (30%) An undamped spring-mass system, of which m_1 is mass of the main body, k_1 the spring constant, and $f(t)$ the applied excitation, is shown in Fig. 4a, this is referred to as the primary system. The equation of vibration by Newton's second law of motion is an ordinary differential equation as

$$\ddot{x}+\omega^2 x = f(t)/m_1 . \qquad \text{(4-1)}$$

where $\omega=\sqrt{k_1/m_1}$.

(a) (12%) Show that by using *the method of variation of parameters* the general solution of equation (4-1) including both the homogeneous and particular solutions is of the form

$$x = A\sin\omega t + B\cos\omega t + \frac{1}{\omega m_1}\int_0^t f(\tau)\sin\omega(t-\tau)\,d\tau , \qquad \text{(4-2)}$$

where A and B are integration constants determined by initial conditions.

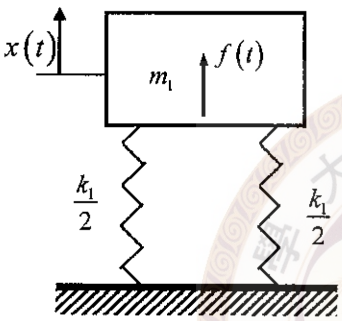

Fig. 4a

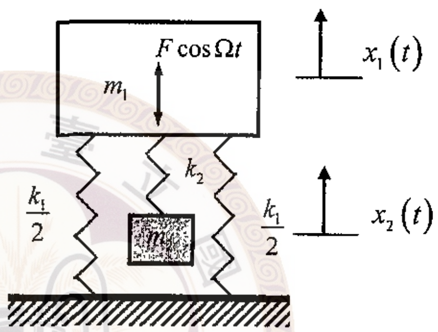

Fig. 4b

(b) (6%) If $f(t)$ is a harmonic excitation given by $f(t)=F\cos\Omega t$ where Ω is the excitation frequency, show that the general solution is in the form

$$x(t) = A\sin\omega t + E\cos\omega t - \frac{F}{m_1\left(\Omega^2-\omega^2\right)}\cos\Omega t , \qquad \text{(4-3)}$$

where A and E are constant.
The third term on the right of equation (4-3) is also called the *steady-state solution.*

(c) (7%) One method of reducing the vibration amplitude of the primary system subjected to harmonic excitation is to attach a tuned vibration absorber, which is a second spring-mass system, as shown in Fig. 4b. The equations of motion of the two degree-of-freedom system, written in matrix form, are

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}\cos\Omega t . \qquad \text{(4-4)}$$

Let the *steady-state solution* be given by

$$x_1 = U_1\cos\Omega t , \qquad x_2 = U_2\cos\Omega t \qquad \text{(4-5)}$$

and show that

$$U_1 = \frac{\left(k_2-m_2\Omega^2\right)F}{D(\Omega)}, \qquad U_2(\Omega) = \frac{k_2 F}{D(\Omega)}, \qquad \text{(4-6)}$$

where

$$D(\Omega) = \left(k_1+k_2-m_1\Omega^2\right)\left(k_2-m_2\Omega^2\right)-k_2^2 .$$

(d) (5%) How should we choose the values of m_2 and k_2 for a given value of Ω so that the amplitude U_1 is reduce to aero?

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