

1. (15%)

Evaluate $\int \frac{1}{\sqrt{e^{2x}-6}} dx$

2. (15%)

For two random variables X and Y with covariance $\text{Cov}(X, Y) = 0$,Find the correlation coefficient $\rho(X+Y, X-Y)$.

3. (15%)

For two random variables X and Y with joint probability density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find $f_{X|Y}(x|y)$

4. (15%)

Solve $x'' + 16x = \cos 4t$ with $x(0) = 0$ and $x'(0) = 1$.

5. (20%)

Solve the partial differential equation $\frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}$ with $-\infty < x < \infty$, $t > 0$ and boundaryconditions: $u(x, 0) = 4e^{-3|x|}$ and $\frac{\partial u}{\partial t}(x, 0) = 0$.

6. (20%)

For $A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

a. Determine the rank of A .b. Determine the nullity of A .c. Find a basis for null space of A .