國立臺灣大學96學年度碩士班招生考試試題

※ 注意：務必依照題號順序作答。

(1) (10%) Let matrices $A=\begin{bmatrix}1&0&0&0\\-2&3&0&0\\0&-4&5&0\\0&0&-6&7\end{bmatrix}$, $I=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ and $B=(I+A)^{-1}(I-A)$,

calculate the matrix $(I+B)^{-1}$.

(2) (10%) If the rank of the set of vectors $\boldsymbol{b}_1=(0,1,-1)$, $\boldsymbol{b}_2=(a,2,1)$, $\boldsymbol{b}_3=(b,1,0)$ is equal to the rank of the set of vectors $\boldsymbol{a}_1=(1,2,-3)$, $\boldsymbol{a}_2=(3,0,1)$, $\boldsymbol{a}_3=(9,6,-7)$ and $\boldsymbol{b}_3$ can be represented as the linear combination of $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, find the values of a, b.

(3) (10%) Given 3×3 matrix A and four vectors $\boldsymbol{a}=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $\boldsymbol{b}=\begin{bmatrix}5\\3\\2\end{bmatrix}$, $\boldsymbol{c}=\begin{bmatrix}1\\3\\-1\end{bmatrix}$, $\boldsymbol{d}=\begin{bmatrix}-2\\2\\-3\end{bmatrix}$

satisfying $A\boldsymbol{a}=\boldsymbol{b}$, $A\boldsymbol{b}=\boldsymbol{c}$, $A\boldsymbol{c}=\boldsymbol{d}$, find $A\boldsymbol{d}$.

(4) (20%) (a) If $a_0=2, a_1=3$, and $a_{n+1}=3a_n-2a_{n-1}$, for all $n\ge 1$, use generating function method to find the formula for a_n.

(b) Redo part (a) using Eigen value method.

(5) (10%) Suppose that $\Re$ is an equivalence relation on $\{1, 2, 3, 4, 5\}$ and the equivalence classes induced by $\Re$ are $\{1, 5\}$, $\{2, 4\}$ and $\{3\}$. What is the value of $|\Re|$, i.e., the size of $\Re$?

(6) (20%) Suppose that $(K, \cdot, +)$ is a Boolean algebra. An element e (or z) of K is called the *identity* (or *zero*) if $e\cdot a=a\cdot e=a$ (or $z+a=a+z=a$) for all $a\in K$. Prove that the identity and zero of K are unique.

(7) (20%) Suppose that $G=(V, E)$ is a connected planar graph. It is known that $|V|-|E|+r=2$ holds for any planar drawing of G, where r is the number of regions. Assume $r>1$. Prove that $|E|\le 3\times|V|-6$ also holds for G. (Hints: every region is bounded by at least three edges and every edge is shared by at most two regions.)

試題隨卷繳回