

1. Consider each of the systems with the input-output difference equation given as follows:

- (1)  $y[n] = x[n] - x[n-1]$
- (2)  $y[n] = \sum_{k=0}^n x[k]$
- (3)  $y[n] = nx[n]$
- (4)  $y[n] = 2x[n-1]$

Answer the following questions:

- (a) (6%) Which of the four systems are linear? You should justify your answer.
  - (b) (7%) Which of the four systems are time-invariant? You should justify your answer.
  - (c) (7%) Find the unit impulse response  $h[n]$  for those systems that are both linear and time-invariant.
2. Consider a system which is a cascade of two linear time-invariant (LTI) systems.
- (a) (8%) Let the first LTI system have the input-output difference equation given by  $y[n] = x[n] + \frac{5}{6}x[n-1]$  and the second LTI system have the system function  $H_2(z) = 1 - 2z^{-1} + z^{-2}$ . Find the system function of the overall cascade system.
  - (b) (12%) Let the first LTI system be a general finite impulse response (FIR) filter. Determine the conditions on the system function  $H_1(z)$  of first LTI system if  $H_2(z)$  is to be a stable and causal inverse filter for  $H_1(z)$ .
3. Consider the following wireless communication technologies: (1) Bluetooth (2) WiMAX (3) UWB (4) HSDPA (5) WiFi
- Answer the following questions:

- (a) (2%) Which one is designed for local area networking?
- (b) (2%) Which one is designed for broadband wireless access over wide range area?
- (c) (2%) Which one is used for "Wireless USB" technology?
- (d) (2%) Which one is an evolutionary improvement for the UMTS-based 3G networks?
- (e) (2%) Which one is designed for low data rate connections over short distances?

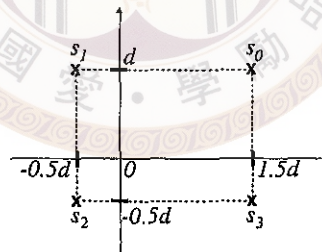


Figure 1: Signal constellation 1

4. Consider the signal constellation in Fig.1 using the following symbol mapping.

Bits: $b_0b_1$	Symbol $s_i$
00	$s_0$
10	$s_1$
01	$s_2$
11	$s_3$

Assume that each bit equals to 0 or 1 with equal probability  $\frac{1}{2}$ . Given that the channel is AWGN with a power spectral density (PSD) of  $\frac{N_0}{2}$ , and the receiver makes the decision based on the minimum distance (MD) decision rule.

- (a) (5%) Derive and express the symbol error probability by  $d$ ,  $N_0$ , and  $Q$ -function (or  $\text{erfc}$ ).
- (b) (5%) Derive and express the bit error probability of the first bit  $b_0$  by  $d$ ,  $N_0$ , and  $Q$ -function (or  $\text{erfc}$ ).
- (c) (5%) Find the  $\frac{E_b}{N_0}$  required to achieve a bit error probability of  $10^{-5}$  for the second bit  $b_1$ , where  $E_b$  denotes the transmitted energy per bit. Note:  $Q(4.265) = 10^{-5}$ .

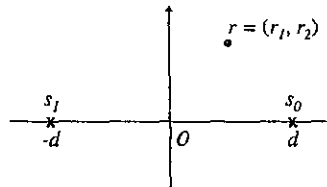


Figure 2: Signal constellation 2

5. Consider the signal constellation in Fig.2 . The probability of  $s_0$  being transmitted is  $p$  and the probability of  $s_1$  being transmitted is  $1 - p$ ,  $p \neq \frac{1}{2}$ . Assume that the receiver knows the symbols are unequally likely to be transmitted and it also knows the exact value of  $p$ . Given that the channel is AWGN with a PSD of  $\frac{N_0}{2}$ , the receiver makes decision based on the maximum a posteriori probability (MAP) decision rule. Let  $r = (r_1, r_2)$  denote the match filter output at the receiver.
- (5%) Derive and express the decision rule of the receiver using  $d$ ,  $N_0$ ,  $r_1$ ,  $s_i$  (not necessarily using all). The decision rule has to be simplified to its simplest form.
  - (5%) Given that  $p = \frac{1}{4}$  and  $\frac{d}{N_0} = 2$ , plot the decision region for each symbol according to the MAP decision rule. You should label your figure clearly and correctly without any ambiguity.
  - (5%) Now consider the signal constellation in Fig. 3. The probability of each symbol being transmitted is given as follows.

$s_i$	$s_0$	$s_1$	$s_2$	$s_3$
$P(s_i)$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{2}{8}$

The channel is AWGN and we assume that the receiver knows the exact values of  $P(s_i)$  and makes decision based on the MAP rule. Given that  $\frac{d}{N_0} = 2$ , plot the decision region for each symbol according to the MAP decision rule. You should label your figure clearly and correctly without any ambiguity.

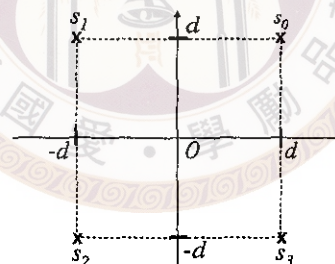


Figure 3: Signal constellation 3

6. Consider a convolutional code with generators  $g_1 = [101]$ ,  $g_2 = [111]$ . Assume that trellis termination is not applied in encoding and the encoder always starts with the zero state.
- (5 %) Explain why convolutional code has the word "convolutional" in its name?
  - (5 %) Please plot the trellis diagram of this code. You should specify the associated input/output bits for each branch.
  - (5 %) Given the demodulator output  $y = 11010011$ , please find the hard decision decoding result for the decoder. You should show the decoding process.
  - (5 %) If BPSK is used for modulation with the signal constellation shown in Fig.4.

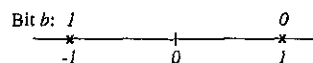


Figure 4: Signal constellation 4

Given the match filter output values of eight BPSK symbols  $r = [0.3, 0.5, -0.1, -0.1, -0.2, -0.5, -0.3, 0.4]$ , please find the soft decision decoding result for the decoder. You should show the decoding process.