1. (20 pts) Show that the difference of any two solutions of the equation

$$\frac{d^2}{dt^2}y + 2\frac{d}{dt}y + 4y = g(t)$$

approaches to zero as t goes to infinity.

2. (20 pts) Find two linearly independent solutions of $t^2\frac{d^2}{dt^2}y = 2y$ of the form $y(t) = t^r$. Using these solutions, find the general solution of $t^2\frac{d^2}{dt^2}y - 2y = t^2$.

3. (20 pts) Solve the initial-value problem as follows:

$$\frac{d^2}{dt^2}y - 2\frac{d}{dt}y + y = \begin{cases} 0 & \text{if } 0 \le t < 1, \\ t & \text{if } 1 \le t < 2, \\ 0 & \text{if } 2 \le t < \infty, \end{cases} \quad (1)$$

$y(0) = 0$, $\frac{d}{dt}y(0) = 1$.

4. (20 pts) Let A be a $n \times n$ symmetric and nonconstant matrix. Give a condition on A such that all solutions of the differential equation $\dot{x} = Ax$ must tend to zero as t goes to infinity. Prove or disprove your answer.

5. (20 pts) Consider the following system of differential equations

$$\begin{cases} \frac{d}{dt}S &= -SI + 2, \\ \frac{d}{dt}I &= SI - 3I, \end{cases} \quad (2)$$

Answer the following questions:

(1) Find all equilibrium solutions of (2). (5 pts)

(2) Are these solutions stable? (7 pts)

(3) Are these solutions asymptotically stable? (8 pts)

Prove or disprove all your answers.