Answers can be given in either English or Chinese.

1. (12%) Given that the three vectors $\alpha_1 = [1,1,0]$, $\alpha_2 = [2,1,0]$ and $\alpha_3 = [1,1,1]$ form a basis for R^3, construct an orthonormal basis for R^3.

2. (12%) Investigate for what values of λ, μ the system

$$\begin{array}{ccccccc} x & + & y & + & z & = & 6 \\ x & + & 2y & + & 3z & = & 10 \\ x & + & 2y & + & \lambda z & = & \mu \end{array}$$

has (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions.

3. (a) (10 %) Find the eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 0 & 2-i \\ 2+i & 4 \end{bmatrix}$. Are the eigenvectors orthogonal?

   (b) (16 %) $\mathbf{A} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ is a diagonalizable matrix.
   (i) Evaluate $e^{\mathbf{A}}$; (ii) If $f(x) = x^4 - 4x^3 + 6x^2 - x - 3$, find $f(\mathbf{A})$.

4. (25 %) $y = y(t)$ is a function of the independent variable t. Solve the following differential equations.
   (a) $\frac{dy}{dt} + y^2 = 1$.
   (b) $(t^2 + t)\frac{dy}{dt} = -(3ty + 2y)$.

5. (25 %) $y = y(t)$ is a function of the independent variable t. Consider the differential equation

$$L[y] = \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - \frac{dy}{dt} - y = e^t \cos t.$$

   (a) Find the general solution of the corresponding homogeneous equation $L[y] = 0$.
   (b) If given the initial conditions $\left.\frac{d^2y}{dt^2}\right|_{t=0} = 6/25$, $\left.\frac{dy}{dt}\right|_{t=0} = 24/25$, and $y(0) = 46/25$, find the solution of the inhomogeneous equation given above.