

1. (10%) Let two vectors  $P = (3,1)$  and  $Q = (1,3)$ . Find  $\cos \theta$  in which  $\theta$  is the angle between these two vectors.
2. (15%) Find a unit vector  $u$  in the direction of  $v = (3,4)$ . Find all the possibilities of a unit vector  $U$  perpendicular to  $u$ .
3. (20%) Find the determinants of rotation  $R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$  and reflection  $Q = \begin{bmatrix} 1-2\cos^2 \theta & -2\cos \theta \sin \theta \\ -2\cos \theta \sin \theta & 1-2\sin^2 \theta \end{bmatrix}$ .
4. (20%) Consider a matrix  $M = \begin{bmatrix} 1 & a \\ c & b \end{bmatrix}$ . (a) What is LU factorization of the matrix? (b) Under what condition is this matrix singular?
5. (20%) Consider an ordinary differential equation  $mx'' + kx = 0$  in which the prime indicates the derivative with respect to  $t$ . (a) Find the general form for the solution  $x(t)$ . (b) Find the solution for  $m = \frac{9}{4}$ ,  $k = 1$  with the initial conditions of  $x(0) = -4$  and  $x'(0) = 0$ .
6. (15%) Given a convex polygon, derive and write down the procedures for obtaining the area of the polygon (a simple convex polygon with six edges is shown below. Nevertheless, your procedures should be general enough for solving any convex polygon with edges greater than three).

