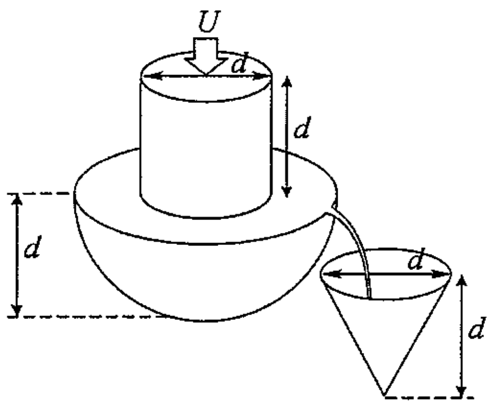

Fig. 1

Problem 1:
A solid cylinder of diameter d and height d is slowly pushed at constant speed U into a hollow hemispheric tank of radius d, filled with water. The overflow is received in a hollow cone of diameter d and height d, which is initially dry. If d and U are known, find the time evolution of the water depth $h(t)$ inside the cone.

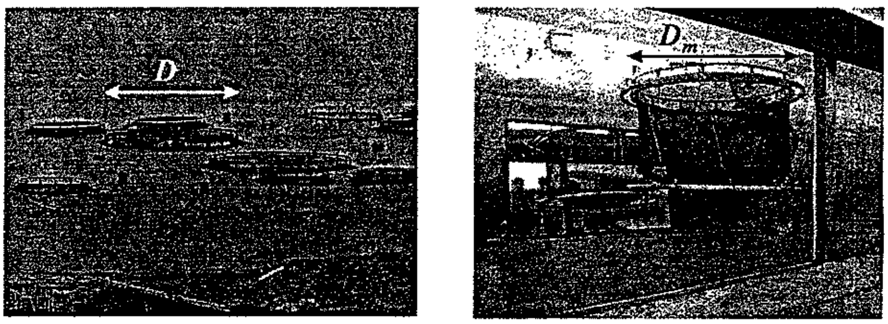

Fig. 2. Fish cage subject to water current. Left: prototype; right: model (Source: THL)

Problem 2. Model tests are performed to find the tension force F needed to anchor a fish cage of diameter D subject to a water current speed V in water of depth H. Water has density ρ and viscosity μ.

a) Using the MLT system, give the units of quantities F, D, V, H, ρ and μ.

b) We expect the tension force F to depend on the other variables according to

$$F = f(D, V, H, \rho, \mu)$$

Choosing D, V and ρ as repeating variables, find a corresponding non-dimensional relationship.

c) If we perform laboratory experiments at scale $D_m / D = 1/20$, using water, how should we choose the depth scale H_m / H and velocity scale V_m / V ?

d) Using these scales, the tension force measured in the laboratory experiments is $F_m = 100$ N. Find the tension force F in the prototype under similar conditions.

e) Give another example of good choice for the repeating variables, and an example of bad choice for the repeating variables.

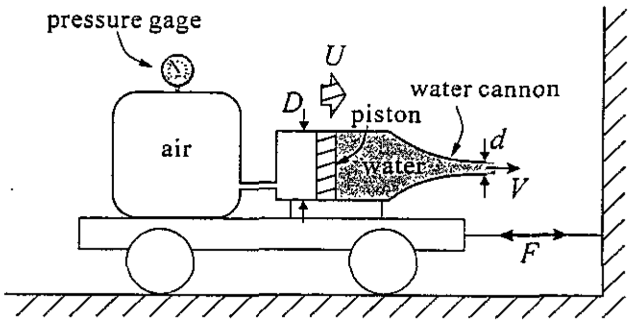

Fig. 3

Problem 3: A water cannon connected to a pressure vessel is mounted on a frictionless carriage, as shown on the figure above. The diameters of the piston chamber and nozzle are $D = 1$ m and $d = 2$ cm. The frictionless piston separating air and water has speed $U = 1$ cm/s. The carriage is linked by a cable to a rigid wall.

a) Find the water velocity V at the nozzle (= mouth) of the cannon.

b) Find the tension force F in the cable.

c) Find the air pressure p at the gage.

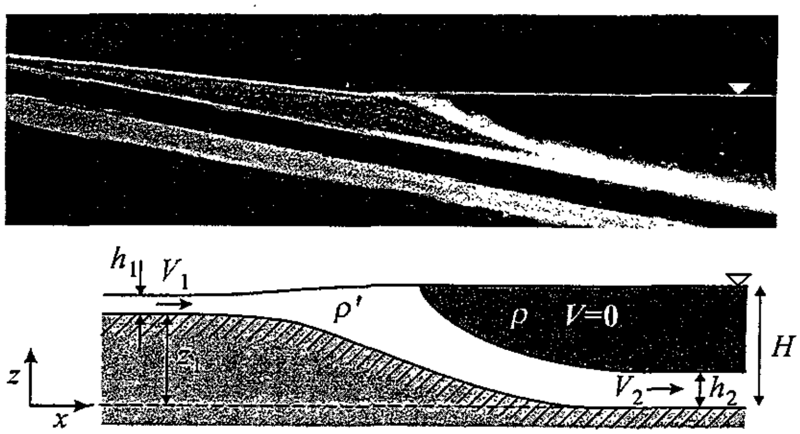

Fig. 4

Problem 4: An open-channel flow of unknown density ρ' enters a lake of depth H, and plunges along the lake bottom as a density current (underflow) underneath a layer of clear water of density $\rho = 1000$ kg/m^3 and velocity $V = 0$. We measure $z_1 = 6$ m, $h_1 = 1$ m, $V_1 =$ 2 m/s, and $h_2 = 2$ m. Assume steady flow and neglect mixing and energy dissipation.

a) Draw the streamlines of the current.

b) Find the lake depth H.

c) Find the underflow velocity V_2.

d) Find the density of the inflow ρ'.

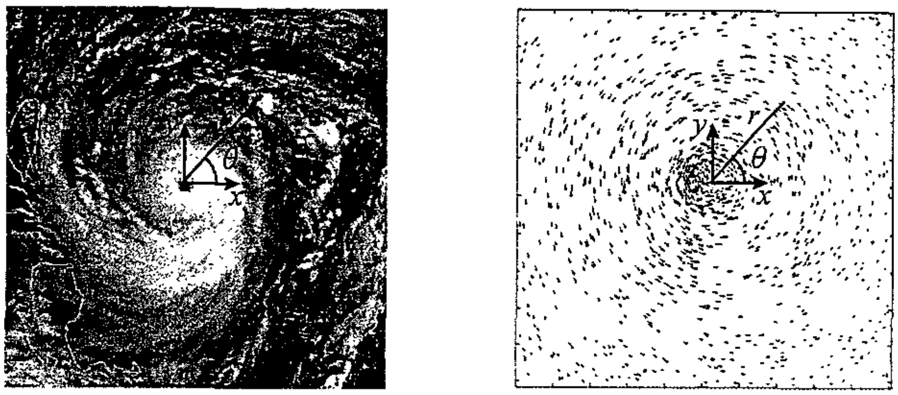

Fig. 5. Typhoon flow. Left: satellite photo of the 1996 Herb Typhoon (source: NOAA). Right: velocity field from lab experiments (source: NTU Institute of Mechanics/Huang S.-Y.)

Problem 5: In polar coordinates, the 2D velocity field due to a typhoon is described by

$$v_r = 0, \qquad v_\theta = \frac{A}{r} + Br$$

a) Express the velocity field in Cartesian coordinates, i.e. find $u(x,y)$ and $v(x,y)$.

b) Find the divergence $\vec{\nabla}\cdot\vec{V}$ (=dilation rate) and the vorticity $\vec{\nabla}\times\vec{V}$ of the flow field.

c) If possible, find the stream function $\psi(r,\theta)$ of the flow.

d) If possible, find the potential $\phi(r,\theta)$ of the flow.

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