1. (15%) Under certain conditions it is found that the rate at which a solid substance dissolves varies directly as the product of the amount of undissolved solid present in the solvent and the difference between the saturation concentration and the instantaneous concentration of the substance. If 10 pounds of solute is dumped into a tank containing 100 pounds of solvent and at the end of 10 minutes the concentration is observed to be 1 part in 20, find the amount of solute in solution at any time t if the saturation concentration is 1 part of solute in 10 parts of solvent.

2. A ball of mass m is thrown vertically downward from a building h feet high. The initial velocity of the ball is v_0. Suppose the air resistance can be neglected.
   (a) (10%) Show that the ball will impact on the ground at time $\left(\sqrt{v_0^2+2gh}-v_0\right)/g$.
   (b) (10%) Suppose ball 1 with m_1=2 pounds is dropped downward from the building with zero initial velocity. After it has fallen k feet (k<h), ball 2 with m_2=4 pounds is dropped downward from the same point with zero initial velocity. Show that, when the first ball hits the ground, the second ball still has $(2\sqrt{hk}-k)$ feet to go.

3. (15%) Consider the initial value problem

$$\frac{d^2x(t)}{dt^2}+x(t)=f(t),\quad t\geq 0,\quad x(0)=\frac{dx(0)}{dt}=0,$$ where f(t)=t for $0\leq t\leq 1$, and f(t)=1 for $1<t$

Find the solution by means of Laplace transforms.

4. Evaluate the line integral $\int_C \vec{F}\bullet d\vec{R}$, where $\vec{F}=[\ln(y)+\cos(x)\cos(y)]\vec{i}+\left[\frac{x}{y}-\sin(x)\sin(y)\right]\vec{j}$ is a vector field and $\vec{R}=x\vec{i}+y\vec{j}$ is the position vector in the x-y plane, for the following cases:

   (a) (10%) C is a path from $(0,\pi/2)$ to $(1,1)$ in the domain y>0.

   (b) (10%) C is a *simple closed path in the domain* y>0.

5. Find the Fourier series representation of the following functions, both defined on [-1,1]:
   (a) (5%) f(x)= -1 for $-1\leq x<0$, and f(x)=1 for $0\leq x\leq 1$
   (b) (5%) $f(x)=\sin(5\pi x)+\cos(3\pi x)$

6. (20%) Solve the problem below:

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial^2 u(x,t)}{\partial x^2},\quad -1<x<1,\quad 0<t$$
$$u(-1,t)=2,\quad u(1,t)=4,\quad 0<t$$
$$u(x,0)=3+x+\sin(2\pi x),\quad -1<x<1$$

試題隨卷繳回