共 9 大題。總分 100 分。

1. 【2*5 points】Show the set of elements that are only in the first set.
   a. {{1, 3, 5}, {2, 4, 6}, {5, 5, 1, 3}} and {{3, 5, 1}, {6, 4, 4, 4, 2}, {6, 4, 4, 2, 6}}
   b. {{5, 3, 5, 1}, {2, 4, 6}, {5, 1, 3, 3}} and {{1, 3, 5, 1}, {6, 4, 2}, {6, 6, 4, 4, 6}}
   c. {{}} and {}
   d. {1, 2}×{a, b} and {1, a}×{2, b}
   e. the power set of {1, 2} and {1, 2, {1, 2}}
2. 【2*2 points】Use a membership table of sets A and B to prove
   a. Absorption Law
   b. Principle of Inclusion-Exclusion
3. 【2*3 points】Use a Venn diagram of sets A, B, and C to prove
   a. Distributive Law for Union
   b. Associative Law for Union
   c. DeMorgan's Law for Union
4. 【5*2 points】Show the combinatorial circuit of
   $(x \wedge y \wedge z) \vee (\neg x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z) \vee (x \wedge y \wedge \neg z)$.
   Prove that it can be simplified to $(x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$.
5. 【5*2 points】Let A be { d, o, m, a, i, n } and B be { c, o, d, o, m, a, i, n }.
   a. How many functions are there with domain A and codomain B?
   b. How many 1-1 functions are there with domain B and codomain A?
6. 【10 points】Find a formula (using the connectives $\wedge$, $\vee$, $\neg$) equivalent to if a then (if b then c else d) else (if e then d else c).
7. 【10 points】Give conditions on sets A and B such that $F \cup G$ is a function for every $F: A \to C$ and $G: B \to C$.
8. 【10 points】Draw the Huffman tree associated with the characters with probabilities P('a')=0.13, P('b')=0.15, P('c')=0.08, P('d')=0.12, P('e')=0.07, P('f')=0.45.
9. 【10*3 points】Prove that
   a. $\{n \mid n \in N \text{ and } \sin(n^* \pi) = 0\} = N$ (natural number)
   b. with 3-dollar and 5-dollar stamps, we can make any amount of postage except 1, 2, 4, and 7 dollars
   c. formula $(a \vee b \vee c \vee d \vee \neg b) \wedge (c \vee d \vee \neg c \vee e \vee f)$ is a tautology.

試題隨卷繳回