(1) **Derive the formal Taylor's series for** $f(x)=ln(x+4)$ about the center at $x_0=2$ (at least 4 terms) and find the region of the radius of convergence for the series. (10%)

(2) **If** H **is the function of** x,y, i.e. $H=H(x,y)$, x is the function of t, i.e. $x=x(t)$, and y is the function of t also, i.e. $y=y(t)$, use chain rule to evaluate $\frac{dH(x(t),y(t))}{dt}$ and $\frac{d^2H(x(t),y(t))}{dt^2}$ (10%)

(3) **Solve the differential equation**

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$$

for the initial conditions $y(0)=1$ and $\frac{dy(0)}{dx}=4$ (10%)

(4) **Using power series method to solve the following differential equation**

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=1+x^2 \qquad (10\%)$$

(5) **Apply Laplace transform to solve the integro-differential equation**

$$\frac{Y(t)}{dt}+5\int_0^t \cos 2(t-u)Y(u)du=10$$

for the initial condition $Y(0)=2$ (10%)

(6) **Show** that 1 and x^2 are orthogonal on [-1, 1] with respect to the weight function $p(x)=x$.

(10%)

(7) **Please solve the following systems, and then indicate the existence of solutions or no solutions by using Rank of matrix. (Please show the details and reasons of your work and solutions)**

(a)
$$\begin{aligned} &2x_1-3x_2+x_4-x_6=0\\ &3x_1-2x_3+x_5=1\\ &x_2-x_4+6x_6=-3 \end{aligned}$$
, (5%)

(b)
$$\begin{aligned} &3x_2-4x_4=10\\ &x_1-3x_2+4x_5-x_6=8\\ &x_2+x_3-6x_4+x_6=-9\\ &x_1-x_2+x_6=0 \end{aligned}$$
, (5%)

(8) **Integrate the given function over the given contour C, counterclockwise or as indicated. (Please show the detail of your work)**

(a) $f(z)=\frac{\sin z}{4z^2-8iz}$, C consists of the boundaries of the squares with vertices ± 3, $\pm 3i$, (counterclockwise) and ± 1, $\pm i$ (clockwise) (5%)

(b) $f(z)=\frac{(1+z)\sin z}{(2z-1)^2}$, $C:|z-i|=2$. (5%)

(9) **Please solve the following boundary value problem.** (20%)

$$\frac{\partial^2 y}{\partial t^2}=9\frac{\partial^2 y}{\partial x^2} \quad for \quad 0<x<4, \quad t>0,$$

$$y(0,t)=y(4,t)=0 \quad for \quad t\geq 0,$$

$$y(x,0)=2\sin(\pi x), \quad \frac{\partial y}{\partial t}(x,0)=0 \quad for \quad 0\leq x\leq 4.$$

試題隨卷繳回