國立臺灣大學97學年度碩士班招生考試試題

1. 某工廠有 A 與 B 兩條製程相同且獨立運作之生產線製造相同之產品。自開工以來，各生產線相鄰兩次當機間隔時距(以日計算)分別如下表所示：

(20%)

A	34	8	253	6	10	62	16	24	48	20	12	103	53	20	200
B	85	59	96	4	63	118	54	36	20	57	31	52	1	17	10

假設當機事件之發生可以帕松歷程(Poisson process)視之，且若兩條生產線之當機間隔時距分別以隨機變數 X_A 與 X_B 表之。

(1) 說明 X_A 與 X_B 之機率密度函數，並以最大概似法(maximum likelihood method)推估 X_A 與 X_B 之機率密度函數參數值。

(2) 推導隨機變數 X_A 與 X_B 之期望值(以機率密度函數之參數表之)。

(3) 估算 90 天內該工廠兩條生產線均不當機之機率。

2. An environmental variable Y varies with time t and can be expressed by the following function:

(20%)

$$Y = \frac{1}{50}t + \varepsilon,\ 0 \le t \le 10$$

where ε is a normally distributed random variable with mean 0.5 and variance 1, i.e. $\varepsilon \sim N(0.5, 1)$.

(1) Calculate the expected value and variance of Y.

(2) What is the probability that the value of Y exceeds 2.0 at time t =10.

3. 某研究人員在一田區隨機選取 22 個位置採取土壤樣本，並測得各土樣重金屬鉻之濃度(以 mg/Kg 計)如下表。假設各樣本之重金屬濃度互相獨立。

(20%)

7.26	5.48	4.83	3.16	5.86	5.32	2.06	8.83	3.59	6.99	4.85
3.55	5.85	4.65	5.86	7.13	3.77	3.95	4.71	6.16	6.76	7.69

依據環保法規，若該田區之平均鉻濃度高於 5mg/Kg，則判定該田區為重金屬污染田區。該研究員擬依據上述採樣資料，以統計檢定(hypothesis test)判定該田區是否受重金屬污染。試問

(a) 虛擬假設(null hypothesis)與替代假設(alternative hypothesis)應如何設定？[請說明理由]

(b) 檢定統計量(test statistic)為何？

(c) 在α=0.05 之顯著水準(level of significance)下，該田區是否應被判定為重金屬污染田區？

4. (1) The weekly demand for a certain drink, in thousands of liters, at a chain of convenience stores is a continuous random variable $g(X) = X^2+X-2$, where X has the density function

(20%)

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & elesewhere \end{cases}$$

Find the expected value for the weekly demand of the drink.

(2) Let X and Y denote the amount of two different types of impurities in a batch of a certain chemical product. Suppose that X and Y are independent random variables with variances $\sigma_X^2 = 2$ and $\sigma_Y^2 = 3$. Find the variance of the random variable $Z = 3X-2Y+5$.

5. The television picture tubes of manufacturer A have a mean lifetime of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean lifetime of 6.0 years and a standard deviation of 0.8 year. What is the probability that a random sample of 36 tubes from manufacturer A will have a mean lifetime that is at least 1 year more than the mean lifetime of a sample of 49 tubes from manufacturer B?

(20%)

見背面

參考公式與數據

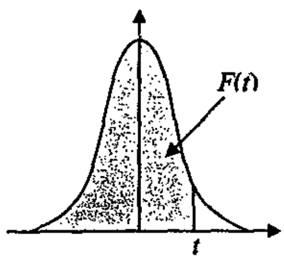
帕松分布機率密度函數

$$f_X(x;\lambda)=\frac{e^{-\lambda}\lambda^x}{x!} \quad x=0,1,2,\cdots$$

指數分布機率密度函數

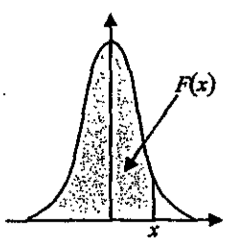
$$f_X(x;\lambda)=\lambda e^{-\lambda x},\ x\ge 0\quad ,\lambda>0.$$

t 分布之累積分布函數(*n*:自由度(degree of freedom)，*F*:累積機率)



n \ F	.75	.90	.95	.975	.99
1	1.000	3.078	6.314	12.706	31.821
2	.816	1.886	2.920	4.303	6.965
3	.765	1.638	2.353	3.182	4.541
4	.741	1.533	2.132	2.776	3.747
5	.727	1.476	2.015	2.571	3.365
21	.686	1.323	1.721	2.080	2.518
22	.686	1.321	1.717	2.074	2.508
23	.685	1.319	1.714	2.069	2.500
24	.685	1.318	1.711	2.064	2.492

標準常態分布之累積分布函數



x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

Table of cumulative probability for normal distribution ($P(Z\le z)=p$)

z	1.05	1.15	1.25	1.35	1.45
p	0.8531	0.8749	0.8944	0.9115	0.9265
z	1.55	1.65	1.75	1.85	1.95
p	0.9394	0.9505	0.9599	0.9678	0.9744
z	2.05	2.15	2.25	2.35	2.45
p	0.9798	0.9842	0.9878	0.9906	0.9929
z	2.55	2.65	2.75	2.85	2.95
p	0.9946	0.9960	0.9970	0.9978	0.9984

試題隨卷繳回