1. Determine whether the following series converge

A. $\sum_{\kappa=1}^{\infty} \kappa^2 e^{-\kappa^3}$. (10 points)

B. $\sum_{\kappa=1}^{\infty} \frac{\kappa+5}{5^{\kappa}}$. (10 points)

2. Solve the following problems

A. Suppose that $f(1) = 2, f(4) = 7, f'(1) = 5, f'(4) = 3$, and f'' is continuous. Find the value of $\int_1^4 x f''(x)\,dx$. (10 points)

B. Differentiate $f(x) = x^{6x} e^{x^2-1}$. (10 points)

C. Find the length of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ between $x = 1$ and $x = 2$. (10 points)

3. Solve the following problems

A. Solve the initial value problem $\frac{dy}{dt} + 2ty = y$, $y(0) = 5$. (15 points)

B. Write the *Taylor* polynomial of degree n of $f(x) = e^x$ centered at -1. What is the equation of the tangent to the graph of f at the point $(-1, e^{-1})$? (15 points)

4.

A. Find an antiderivative $G(x)$ of the function $\ln(x)$. (Hints: use an integration by parts) (10 points)

B. Find all functions $y(x)$ satisfying the differential equation $y' + 2xy = \ln(x)e^{-x^2}$.

(If you haven't answered the first equation, write $G(x)$ for an antiderivative of $\ln(x)$) (10 points)

試題隨卷繳回