※ 注意：可用不具附有儲存程式功能之計算器　（每題 20 分）

1. Consider a separately excited dc motor shown in Fig. 1. Define state variables as $x_1(t) = i(t)$, $x_2(t) = \omega(t)$, $x_3(t) = \theta(t)$, output variable as $y(t) = x_3(t)$, and control input as $u(t)$.

   Questions: (1) find the state equation; (2) find the transfer function by Laplace transform.

2. Consider a dynamical system modeled by the following nonlinear state equation:

   $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, u) = [f_1(\mathbf{x}, u) \quad f_2(\mathbf{x}, u) \quad f_3(\mathbf{x}, u)]^T$ where $f_1(\mathbf{x}, u) = x_2(t) + u^2(t)$,

   $f_2(\mathbf{x}, u) = \frac{1}{2} x_3^2(t)\sin(2x_1(t)) - \sin x_1(t) - x_2(t)$, $f_3(\mathbf{x}, u) = \cos x_1(t) - 1$. Let u_e be a constant input that forces the system to settle into a constant equilibrium state $\mathbf{x}_e$.

   Question: find a linear approximation of the state equation about the equilibrium state.

3. *Consider a discrete-time linear time-invariant system modeled by*

   $\mathbf{x}(t+1) = -\mathbf{A}\mathbf{x}(t),\ \mathbf{x}(t_0) = \mathbf{x}_0$

   Lyapunov approach to the stability analysis of the system can be done by evaluating the quadratic positive definite form

   $V(\mathbf{x}(t)) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t)$

   Questions: to assure the asymptotical stability, (1) derive the so called discrete-time Lyapunov equation; (2) show the necessary and sufficient condition on the matrix **P**.

4. Consider the PD feedback control system shown in Fig. 2.

   Questions: (1) find the gain constant K so that the relative damping ratio of the closed-loop system is 0.5; (2) find the bandwidth of the resulting system.

5. Consider a continuous-time linear time-invariant system modeled by the following state equation:

   $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}1 & -3\\ 5 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t),\ u(t) = -[K_1 \quad K_2]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

   Question: show the region in the K_2 versus K_1 plane in which the overall system is stable (K_1 and K_2 are real constants)

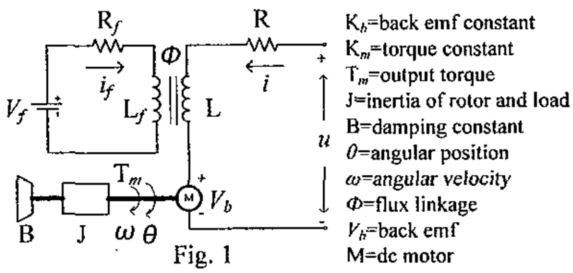

Fig. 1

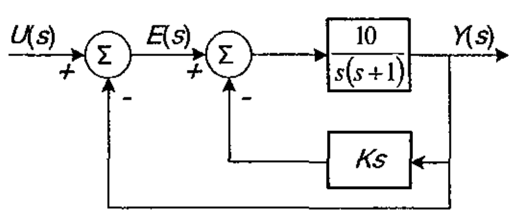

Fig. 2

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