

Problem 1: (12%)

A random process is defined by $X(t) = A \cos(2\pi f_0 t + \Theta)$, where Θ is a random variable uniformly distributed on $[0, 2\pi)$. Find the Autocorrelation and Power spectral density (PSD) of the process:

$$Z(t) = X(t) + \frac{d}{dt} X(t).$$

Problem 2: (16%)

For a AWGN channel, let the transmitted power per sample be P , the noise PSD be $N_0/2$ and the transmission bandwidth be W ,

- (a) state the available channel capacity. (8%)
- (b) give an explanation (e.g., graphical or vector or geometrical explanation) of the channel capacity. (8%)

Problem 3: (12%)

- (a) What is the waveforms of an M-ary QAM modulated signal? (6%)
- (b) Draw the signal constellation for the 16-QAM and specify the signal parameters as precisely as possible. (6%)

Problem 4: (10%)

The carrier $c(t) = 100 \cos(2\pi 10^6 t)$ is frequency modulated by the signal $m(t) = 2 \cos(2000\pi t)$. The deviation constant is $k_f = 2000 \text{ Hz/V}$.

- (a) Determine the resultant bandwidth using the Carson's rule. (5%)
- (b) Plot the spectrum of the modulated signal within the bandwidth. (5%)

Problem 5: (16%)

Assume that the relationship between the input signal $x(t)$ and output signal $y(t)$ of a linear time-invariant (LTI) system is given by

$$y(t) = \int_{-\infty}^t e^{-2(t-\alpha)} x(\alpha - 1) d\alpha$$

- (a) Find the impulse response $h(t)$ of this system. (6%)
- (b) Is this system causal? Why? (5%)
- (c) Is this system stable? Why? (5%)

Problem 6: (12%)

- (a) Consider an LTI system with the input signal $x(t) = 2\cos(4t)$ and the steady-state output signal $y(t) = 5\cos(4t - \pi/4)$. Find the system transfer function $H(s)$. (6%)
- (b) A discrete-time signal is given by $x[n] = a^n u[n] - b^{2n} u[-n-1]$, where $u[n]$ is the unit step function. Find the bilateral z-transform and region of convergence of $x[n]$. (6%)

Problem 7: (10%)

Consider a discrete-time LTI system with the impulse response $h[n] = 3$, for $-1 \leq n \leq 2$, and $= 0$, otherwise. We input a signal $x[n] = 2$, for $1 \leq n \leq 3$, and $= 0$, otherwise, into the system. Let the output signal be $y[n]$.

- (a) Find the output signal $y[n]$ at $n = 5$. (5%)
- (b) Find the maximum value of $y[n]$. (5%)

Problem 8: (12%)

The relationship between the input signal $x[n]$ and output signal $y[n]$ of a discrete-time LTI system is given by $y[n] - 5y[n-1]/6 = x[n]$. Suppose that the input signal is given by $x[n] = 2^n u[n]$ and the initial condition is given by $y[-1] = 0$, where $u[n]$ is the unit step function. Find the corresponding output signal $y[n]$.