第 1 題到第 2 題爲複選題，每題 5 分，請作答於「選擇題作答區」，需完全答對才有分數，答錯不倒扣。第 3 題到第 12 題，請作答於「非選擇題作答區」。

1. A semi-infinite plate coincides with the region $0 \le x \le \pi$ and $y \ge 0$. The right and left edges of the plate are insulated and the bottom end is held at temperature of $f(x)$. Find the suitable differential equation and boundary conditions for the steady state temperature $u(x,y)$: (A) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0<x<\pi$ and $y>0$, (B) $\left.\frac{\partial u}{\partial x}\right|_{x=0}=0$ for $y>0$, (C) $\left.\frac{\partial u}{\partial y}\right|_{x=\pi}=0$ for $y>0$, (D) $u(x,0)=f(x)$ for $0 \le x \le \pi$, (E) $u(\pi,y)=0$ for $y>0$. (5%)
2. The solution of the initial value problem: $(x^2+1)y''+2xy'=0$, $y(0)=0$ and $y'(0)=1$ has the form $y=\sum_{n=0}^{\infty} a_n x^n$. Which of the following items are correct? (A) $a_0=0$, (B) $a_1+a_3=2/3$, (C) $a_2+a_4=1/3$, (D) $\sum_{n=0}^{\infty} a_{2n}=0$, (E) none of above. (5%)
3. Please solve the following differential equation $\left(-2xy+\cos x+\frac{1}{1+y^2}\right)\frac{dy}{dx}=(y+\sin x)y$ with boundary condition $y(0)=1$. (10%)
4. Please solve the following differential equation system: (15%)
$\frac{dx}{dt}=3x-y-z$, $\frac{dy}{dt}=x+y-z$, $\frac{dz}{dt}=x-y+z$ with $x(0)=5$, $y(0)=4$ and $z(0)=4$.
5. (a)Find the differential equations relating R, L_1, L_2, i_2, i_3 and E in Fig. P.5. (b) If R=10Ω, L_1=0.01Henry, L_2=0.0125Henry, E=150V, i_2(0)=0 and i_3(0)=0, find i_1(t), i_2(t) and i_3(t).(15%)

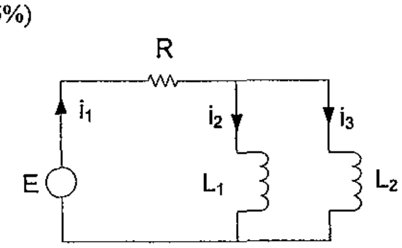

Fig. P.5

見背面

A semiconductor wafer has M VLSI chips on it and these chips have the same circuitry. Each VLSI chip consists of N interconnected transistors. A transistor may fail (not function properly) with a probability p because of its fabrication process, which we assume to be independent among individual transistors. A chip is considered a failure if there are n or more transistor failures. Let K be the number of failed transistors on a VLSI chip, which is therefore a random variable (R.V.).

6. What is a random variable? (5%)
7. What is the sample space (also called outcome set) over which R.V. K is defined? (3%)
8. Let $X_i = 1$ if a chip i fails and $X_i = 0$ if a chip i is good. Derive the probability that a chip is good, i.e., $p_g \equiv \Pr\{X_i = 0\}$ =? (5%)
9. Now suppose that the value of a current I of the chip depends on transistor 1. If transistor 1 fails, we will observe an abnormal I value with a probability p and a normal I value with a probability $1-p$; if transistor 1 is good, we will observe an normal I value with a probability q and an abnormal I value with a probability $1-q$. What is the probability that you observe an abnormal I value? When the I value you measured is normal, what is the probability that transistor 1 actually fails? (8%)
10. Whether one chip is good or fails is independent of other chips. Let the yield of a wafer be defined as the percentage of good chips in the wafer, i.e., $Y = (1 - \frac{1}{M}\sum_{i=1}^{M} X_i) \times 100\%$. Then derive $\mu_Y \equiv$ E[Y]=? σ_Y^2= Var[Y]=?
    (Hint: Utilize p_g obtained from 8)) (10%)
11. Note that M>>100, so you apply the Central Limit Theorem to approximate the probability density function $f_Y(y)$ of R.V. Y by a normal distribution, $N(\mu_Y, \sigma_Y^2)$. Describe Central Limit Theorem and how you apply it. (9%)
12. Let Y_1 and Y_2 be the yields of two semiconductor factories with probability density functions $f_{Y_1} \approx N(\mu_{Y_1}, \sigma_{Y_1}^2)$ and $f_{Y_2} \approx N(\mu_{Y_2}, \sigma_{Y_2}^2)$. Let $Z = Y_1 - Y_2$. Find the probability distribution function of Z, i.e., $F_Z(z)$ =? (10%)

試題隨卷繳回