(15%)
1) Evaluate the integral $\int \dfrac{\log_2 x}{x} dx$.

(15%)
2) State the **central limit theorem** as precise as possible.

(20%)
3) Suppose that children are born at a Poisson rate of 10 per day in a certain hospital. What is the probability that a) at least two babies are born during the next 6 hours; b) no babies are born during the next two days?

(15%)
4) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(x-3)^n}{2^n\, n}$

(15%)
5) Find the solution of the following differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0 \quad \text{with initial conditions} \quad x(0)=1 \quad \text{and} \quad x'(0)=1$$

(20%)
6) Find a normalized QR decomposition of the following matrix, where Q is orthogonal and R is upper triangular.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & -1 & 3 & 2 \\ 1 & -1 & 3 & 2 \\ -1 & 1 & -3 & 1 \end{bmatrix}$$

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