1. Please solve following differential equation systems and show the details of your works.

(1) $\begin{aligned} y_1' &= -3y_1 - y_2 + 2y_3 \\ y_2' &= \quad -4y_2 + 2y_3 \\ y_3' &= \quad\ y_2 - 5y_3 \end{aligned}$ (10%)

(2) $\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= 4y_1 + y_2 \end{aligned}$; $y_1(0) = 4$, $y_2(0) = 4$ (10%)

2. Please solve the following boundary value problem using separation of variables. (20%)

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y}{\partial x^2} \quad for \quad 0 < x < 4, \quad t > 0,$$

$$y(0,t) = y(4,t) = 0 \quad for \quad t \geq 0,$$

$$y(x,0) = 2\sin(\pi x), \quad \frac{\partial y}{\partial t}(x,0) = 0 \quad for \quad 0 \leq x \leq 4.$$

3. Please evaluate

$\oint_C \frac{dz}{z^2+1}$, C: (a) $|z+i| = 1$ (5%); (b) $|z-i| = 1$ (5%)

4. A tank contains 1000 gal of water in which 100lb of salt is dissolved. Two gallons of fresh water runs in per minute and 2 gal of the mixture in the tank, kept uniform by stirring, runs out per minute. How much salt is left in the tank after 6 hours? (25%)

5. If an iron ball of weight W = 50 nt stretches a spring 1.568 m, how many cycles per minute will this mass-spring system execute? What will its motion be if we pull down the weight an additional 12 cm and let it start with zero initial velocity? (25%)

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