1. (10%) Show that the series $\sum_{n=0}^{\infty}\frac{1}{2n+1}$ diverges.

2. (10%) The population of a town is increasing at the rate of $400te^{0.02t}$ people per year, where t is the number of years from now. Find the total gain in population during the next 5 years.

3. (10%) Find the solution of $y'+(\cos x)y=\cos x$, $y(0)=2$.

4. (20%) Suppose x_1 and x_2 are bivariate normally distributed and the definitions of corresponding probability density functions are as follows.

$$f(x_1)=\frac{1}{\sqrt{2\pi\sigma_1^2}}e^{-\frac{(x_1-\mu_1)^2}{2\sigma_1^2}}$$

$$f(x_2)=\frac{1}{\sqrt{2\pi\sigma_2^2}}e^{-\frac{(x_2-\mu_2)^2}{2\sigma_2^2}}$$

$$f(x_1,x_2)=\frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2(1-\rho^2)}}e^{-\frac{1}{2(1-\rho^2)}\left\{\frac{(x_1-\mu_1)^2}{\sigma_1^2}-2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}+\frac{(x_2-\mu_2)^2}{\sigma_2^2}\right\}}$$

a. (5%) Calculate $\int_{-\infty}^{\infty}e^{cx_1}f(x_1)dx_1$.

b. (5%) According to the definition of the conditional probability density function,

$$f(x_2\mid x_1)=\frac{f(x_1,x_2)}{f(x_1)},$$

show that

$$f(x_2\mid x_1)=\frac{1}{\sqrt{2\pi\sigma_2^2(1-\rho^2)}}e^{-\frac{\left(x_2-\left(\mu_2+\rho\frac{\sigma_2}{\sigma_1}(x_1-\mu_1)\right)\right)^2}{2\sigma_2^2(1-\rho^2)}}.$$

c. (10%) Calculate $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{x_1}e^{cx_2}f(x_1,x_2)dx_1dx_2$.

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5. (10%) If f is differentiable at some number z, show that, for any positive numbers a and b with $a < b$,

$$f'(z) = \lim_{h\to 0} \frac{a^2 f(z+bh) - b^2 f(z+ah) + (b^2 - a^2) f(z)}{(a^2 b - b^2 a)h}$$

6. (10%) Compute the line integral:

$$\int_C \frac{(x^3 + x^2 + x + 1)}{x^4} dx$$

where C is the lower quarter-circle centered at 0 joining $\frac{-1-i}{\sqrt{2}}$ and $\frac{1-i}{\sqrt{2}}$ in the positive (counterclockwise) sense.

7. (10%)

a. (5%) Evaluate the following integral:

$$\int_1^2 \int_z^{z+2} (x + 2z^2) dx dz$$

b. (5%) Switch the order of x and z in the above integrals, i.e., rewrite the above integral into a summation of terms of the form (You don't have to evaluate the integral).

$$\int_0^{\square} \int_{\square}^{\square} (x + 2z^2) dz dx$$

8. (20%) Find the limit:

a. (10%)

$$\lim_{h\to 0} \frac{-3h^3 + 8h^2 - 7h + 11}{4h^3 - h^2 + 5}$$

b. (10%)

$$\lim_{z\to 0} \frac{z^2 + 2\cos z - 2}{z^4}$$

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