1. Consider a matrix equation $A = BC$, Find matrix C, if

$$A = \begin{bmatrix} 1 & 2 & 7 & 4 \\ 5 & 6 & 15 & 8 \\ 9 & 10 & 23 & 12 \end{bmatrix} \; ; \; B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

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2. Find the solution of the problem:

$y'' - 6y' + 9y = 5e^{3x}$; $y(0) = 0 ; y'(0) = 0$

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3. Find the Fourier series of the following periodic functions :

$f(x) = \sin x$; when $0 \le x \le 2\pi$ and $f(x) = f(x + 2\pi)$

$g(x) = \sin x$; when $0 \le x \le \pi$ and $g(x) = g(x + \pi)$

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4. If the general solution of the equation $xy'' + 2(1-x)y' + (x-2)y = 0$ is $y(x) = C_1 y_1(x) + C_2 y_2(x)$, and $y_1(x) = e^x$, Find $y_2(x)$

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5. Find the Laurent series of the complex function $f(z) = [z^2 - 3z + 2]^{-1}$ with center $z_0 = 0$, which is valid in the domain $1 < |z| < 2$

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6. Consider one dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty ; t > 0 --- (1)$$

I.C. $\begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$ $\quad -\infty < x < \infty --- (2a,b)$

the D' Alembert' s solution is : $u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)ds$

If the initial conditions $f(x)$ and $g(x)$ are given as:

$f(x) = \begin{cases} 1 & when\ -1 \le x \le 1 \\ 0 & otherwise \end{cases}$; $g(x) = \begin{cases} 1 & when\ -1 \le x \le 1 \\ 0 & otherwise \end{cases}$

Find $u(x,t)$ at $x = 3$, and $t = 3$, assume $c = 1$

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7. Consider the following eigenvalues problem:

$y'' + \lambda y = 0 \quad a \le x \le b \quad --- (1)$

B.C. $\begin{cases} y(a) = y(b) \\ y'(a) = y'(b) \end{cases}$ $--- (2a,b)$

if λ_m and λ_n are two different eigenvalues, show that their corresponding eigenfunctions $y_m(x)$ and $y_n(x)$ are orthogonal in (a,b).

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試題隨卷繳回