1. (20 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function.

   (i) Must f be differentiable on $[0, 1]$? (10 pts)

   (ii) Can $\lim_{x \to 0} f(x)$ exist? (10 pts)

   Prove or disprove your answers.

2. (20 pts) Let $g : [0, 1] \to \mathbb{R}$ be a continuous function. Define
$$G(x) = \int_0^x g(t)\, dt, \quad \forall x \in [0, 1].$$
   Can G be differentiable on $(0, 1)$? Prove or disprove your answer.

3. (20 pts)

   (i) Calculate the following integrals: (10pts)
$$\int_0^t \int_0^{t_1} dt_2\, dt_1 = ?, \quad \int_0^t \int_0^{t_1} \int_0^{t_2} dt_3\, dt_2\, dt_1 = ?$$

   (ii) Find the general formula to
$$\int_0^t \int_0^{t_1} \cdots \int_0^{t_{n-2}} \int_0^{t_{n-1}} dt_n\, dt_{n-1} \cdots dt_2\, dt_1 = ?$$
   for all $n \in \mathbb{N}$. (10 pts)

4. (20 pts) Let $f(x, y, z) = xyz$ for $x, y, z \in \mathbb{R}$ and $S = \{(x, y, z) : x^2 + y^2 = 1, x = z\}$. Find all minimum and maximum of the function f on the set S.

5. (20 pts) Let
$$K(x) = |x|^{-3} \left(1 - 3 \left\langle \frac{x}{|x|}, e_3 \right\rangle^2 \right),$$
   for $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, where $e_3 = (0, 0, 1)$, $|x|^2 = \sum_{j=1}^3 x_j^2$, and $\langle x, y \rangle = \sum_{j=1}^3 x_j y_j$ for $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$.

   (1) (10 pts) Calculate
$$\lim_{\delta \to 0^+} \int_{\{x \in \mathbb{R}^3 : |x| > \delta\}} K(x)\, dx = ?$$

   (2) (10 pts) Calculate
$$\int_{\mathbb{R}^3} K(x)(w \cdot x)\, dx = ?$$
   for all $w = (w_1, w_2, w_3) \in \mathbb{R}^3$.

試題隨卷繳回