1. (7%) (8%) Find the following limits:
(1a) $\lim_{x\to 0} f(x)$ with $f(x) = \sin(\frac{1}{x})I(x > 0)$. (1b) $\lim_{x\to\infty} x\tan(\frac{1}{x})$.

2. (7%) (8%) Find the derivatives of y with respect to x.
(2a) $x^3 + y^2 = \sin^2(y)$. (2b) $y = \tan^{-1}(\sqrt{x^2-1}) + \csc^{-1}(x)$, $x > 1$.

3.(7%) (8%) Evaluate the following definite and indefinite integrals.
(3a) $\int_0^2 \frac{\log_2(x+2)}{x+2}dx$. (3b) $\int \frac{e^{\sin^{-1}(x)}}{\sqrt{1-x^2}}dx$.

4. (15%) Find the volume of the ellipsoid generated by resolving the semiellipse $y = \frac{1}{a}\sqrt{a^2-x^2}$, $|x| \le a$, about the x-axis.

5. (10%) Solve the differential equation $\frac{dy}{dx} = r(M-y)y, 0 < y < M$.

6. (10%) Evaluate $\int\int_D \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}dxdy$, where $D = \{(x,y) : 1 \le x^2+y^2 \le e\}$.

7. (10%) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

8. (10%) Determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln(n)}$. Give reasons for your answer.

試題隨卷繳回